Dynamic output feedback control of discrete-time switched affine systems

Lucas N. Egidio, Grace S. Deaecto, *IEEE Member*

***Abstract*—This paper deals with the co-design of an output-dependent switching function and a full order affine filter for discrete-time switched affine systems. More specifically, from the measured output, the switched filter has the role of providing essential information for the switching function, which must assure global practical stability of a desired equilibrium point. The design conditions are based on a general quadratic Lyapunov function and are expressed in terms of linear matrix inequalities. Moreover, whenever the system is quadratically detectable, the solution to the output feedback problem coincides with the one for the state feedback case and the associated filter admits the observer form. To the best of the authors' knowledge, this is the first time that the dynamic output feedback control problem is treated in the context of discrete-time switched affine systems. The results can be used to cope with sampled-data control and have the property of assuring global asymptotic stability when the sampling period tends to zero. A practical application concerning the velocity control of a DC motor driven by a buck-boost converter illustrates the theoretical results.**

***Index Terms*—Output feedback control, Switched affine systems, Discrete-time domain, Linear matrix inequalities.**

I. Introduction

In the context of hybrid systems, switched systems are an important subclass that arouses the interest of researchers from several engineering fields, as for instance, power electronics [1], [2], [3], [4], networked control systems [5], automotive control [6], and others. These systems are composed of a set of subsystems and a switching rule (also called switching function) responsible for electing one of them as active at each instant of time. This rule can be a perturbation signal or a control variable that is crucial to assure stability and performance enhancement. More discussions within this topic can be found in [7], [8] and [9].

Considering the subclass of switched linear systems, the literature presents a significant number of results regarding state and output feedback control design for both time domains, as for instance, [10], [11], [12]. These systems present the origin as the common equilibrium point of all subsystems, which simplifies considerably the control design problem to be faced. A more general subclass is formed by the switched affine systems, which are more intricate as they possess several equilibrium points composing a region in the state-space. Generally, the equilibrium point of interest is not common to those of the subsystems and, consequently, asymptotic stability of this point is not possible for a limited switching frequency. In the continuous-time domain, results regarding the design of state or output-dependent switching functions assuring global asymptotic stability are available, as for instance, in [13], [14], [15]. In order to avoid high switching frequency, making the control technique more amenable for practical applications, results assuring practical stability are available in the literature, such as [2], [16], [17], [18], [19], [20], [21], and [22]. The three first references are recent and make clear the importance of establishing a dwell-time as a manner of bounding the switching frequency. All of them treat the state feedback control design, exclusively, and only the last three references deal with the problem in the discrete-time domain. More recently, reference [23] has treated global asymptotic stability of a limit cycle, but also only for the state feedback case. To make the problem more realistic, it is important to consider that the state is generally not available and the controller must take into account only a measured output.

This paper addresses dynamic output feedback control design of switched affine systems in the discrete-time domain, assuring global practical stability of a desired equilibrium point. To the best of the authors' knowledge, this problem has never been treated in the literature to date. The proposed conditions are based on a general quadratic Lyapunov function, adopted for the first time in [20], and are expressed in terms of linear matrix inequalities (LMIs) being, therefore, simple to solve by means of readily available tools. More specifically, these conditions allow designing simultaneously a full order switched affine filter and an output-dependent switching function to assure practical stability, guaranteeing that the state trajectories globally converge to a suitable set of attraction. The role of the switched filter is to provide, from the measured output, important information for the switching function, since the state vector is not available. It is demonstrated that whenever the system is quadratically detectable, the solution to the output feedback problem coincides with the one to the state feedback case proposed in [20]. Moreover, the associated switched filter admits the observer form. Two objective functions are of great interest. The first one consists in minimizing the volume of an ellipsoidal set of attraction, which leads to the minimization of a nonlinear but convex objective function. Supposing that the designer is interested in optimizing a specific controlled output, instead of the full state variable, the second problem takes into account the

This research was supported by the "Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES)", Finance Code 001, "National Council for Scientific and Technological Development (CNPq)", under grant number 303499/2018-4 and by São Paulo Research Foundation (FAPESP) under grant number 2017/20343-0.

Lucas N. Egidio and Grace S. Deaecto are with the School of Mechanical Engineering, UNICAMP, 13083-860, Campinas, SP, Brazil {egidio,grace}@fem.unicamp.br. Lucas N. Egidio is also with the Department of Electrical Engineering in Linköping University, 581 83, Linköping, Sweden.

minimization of an upper bound on the Euclidean norm of this controlled output in steady state. The proposed methodology can be used to deal with sampled-data control of continuous-time switched affine systems and has the property of assuring global asymptotic stability in the limit situation in which the sampling period tends to zero. The theoretical results are validated in the velocity control of a DC motor driven by a DC-DC power converter. This is a common arrangement in the industry being the focus of many works in recent years, see [24] and [4].

The notation used throughout is standard. For square matrices, $\mathrm{Tr}(\cdot)$ denotes the trace function. For real vectors or matrices, $(')$ refers to their transpose. For symmetric matrices, $(\bullet)$ denotes each of their symmetric blocks. The symbols $\mathbb{R}$, and $\mathbb{N}$ denote the sets of real and natural numbers, respectively. The set $\mathbb{K} = \{1, \cdots, N\}$ is composed of the N first positive natural numbers. For any symmetric matrix, $X > 0$ $(X \geq 0)$ denotes a positive (semi)definite matrix. The unitary simplex is composed of all nonnegative vectors $\lambda \in \mathbb{R}^N$ such that $\sum_{j\in\mathbb{K}} \lambda_j = 1$ and is denoted by Λ. The convex combination of matrices $\{X_1, \cdots, X_N\}$ is given by $X_\lambda = \sum_{i\in\mathbb{K}} \lambda_i X_i,\ \lambda \in \Lambda$. A square matrix is said to be Schur stable if its eigenvalues lie in the open region $|z| < 1$ of the complex plane.

II. Problem formulation and preliminaries

Consider the discrete-time switched affine system given by

$$\begin{cases} x[k+1] = A_\sigma x[k] + b_\sigma,\ x[0] = x_0 \\ y[k] = C_\sigma(x[k] - x_e) \\ z[k] = E(x[k] - x_e) \end{cases} \tag{1}$$

where $x[k] \in \mathbb{R}^{n_x}$ is the state, assumed not available, $x_e \in \mathbb{R}^{n_x}$ is the equilibrium point of interest, $y[k] \in \mathbb{R}^{n_y}$ is the measured output, $z[k] \in \mathbb{R}^{n_z}$ is the performance output and $\sigma[k] \in \mathbb{K}$ is the switching function responsible for selecting the subsystem to be activated at each instant of time $k \in \mathbb{N}$. In our context, this function $\sigma(y) : \mathbb{R}^{n_y} \to \mathbb{K}$ is a control variable to be designed to govern the state trajectories $x[k]$ towards a set of attraction $\mathcal{X}$, which contains the desired equilibrium point x_e. As discussed in [20], in the discrete-time domain, asymptotic stability is generally impossible to be guaranteed, since the desired point x_e is not necessarily an equilibrium for any subsystem. Thus, the steady state response is enhanced by optimizing some measure of the set of attraction.

It is supposed that the equilibrium point x_e of interest must belong to the set of attainable ones

$$X_e = \{x_e \in \mathbb{R}^{n_x}\ :\ x_e = (I - A_\lambda)^{-1} b_\lambda,\ \lambda \in \mathcal{S}\} \tag{2}$$

with $\mathcal{S} \subseteq \Lambda$ composed of all $\lambda \in \Lambda$ such that A_λ is Schur stable. This set is the same adopted in several works dealing with stabilization of discrete-time switched affine systems, see [20], [22]. Defining the auxiliary state variable $\xi[k] = x[k] - x_e$, we obtain an equivalent affine system given by

$$\begin{cases} \xi[k+1] = A_\sigma \xi[k] + \ell_\sigma,\ \xi[0] = \xi_0 \\ y[k] = C_\sigma \xi[k] \\ z[k] = E\xi[k] \end{cases} \tag{3}$$

with $\ell_i = (A_i - I)x_e + b_i, \forall i \in \mathbb{K}$. Notice that $\ell_\lambda = 0$ whenever $\lambda \in \Lambda$ is associated with $x_e \in X_e$. For this translated system the desired equilibrium point is always the origin $\xi = 0$, which represents $x = x_e$ in the original system (1).

In order to design an output-dependent switching function, let us consider the full order switched affine filter with state-space realization

$$\hat{\xi}[k+1] = \hat{A}_\sigma \hat{\xi}[k] + \hat{B}_\sigma y[k] + \hat{\ell}_\sigma,\ \hat{\xi}[0] = \hat{\xi}_0 \tag{4}$$

where $\hat{\xi} \in \mathbb{R}^{n_x}$ is the filter state variable and the matrices $(\hat{A}_i, \hat{B}_i, \hat{\ell}_i),\ i \in \mathbb{K}$ have to be determined. As it will be clear afterward, the filter (4) provides essential information for the switching function, which is designed to depend only on the measured output $y \in \mathbb{R}^{n_y}$ through $\hat{\xi} \in \mathbb{R}^{n_x}$. Defining the augmented state variable $\tilde{\xi} = [\xi'\ \hat{\xi}']'$ and connecting the filter (4) to the system (3), we obtain

$$\begin{cases} \tilde{\xi}[k+1] = \tilde{A}_\sigma \tilde{\xi}[k] + \tilde{\ell}_\sigma,\ \tilde{\xi}[0] = \tilde{\xi}_0 \\ z[k] = \tilde{E}\tilde{\xi}[k] \end{cases} \tag{5}$$

with matrices

$$\tilde{A}_i = \begin{bmatrix} A_i & 0 \\ \hat{B}_i C_i & \hat{A}_i \end{bmatrix},\ \tilde{\ell}_i = \begin{bmatrix} \ell_i \\ \hat{\ell}_i \end{bmatrix}, \tilde{E}' = \begin{bmatrix} E' \\ 0 \end{bmatrix},\ i \in \mathbb{K} \tag{6}$$

The idea is to design the switching function $\sigma(\hat{\xi})$ in order to assure practical stability of the origin $\tilde{\xi} = 0$ by optimizing some measure related to the augmented set of attraction $\tilde{\mathcal{X}}$. To this end, we propose two objective functions. The first one is the volume minimization of the ellipsoidal set $\mathcal{X} \supset \tilde{\mathcal{X}}$, which is the projection of $\tilde{\mathcal{X}}$ in the subspace generated by the system state variable. This problem consists in minimizing a nonlinear convex function subject to convex constraints, which is well adapted to be solved by means of the Frank-Wolfe algorithm, see [25]. The second objective function is based on the minimization of an upper bound on the Euclidean norm of the controlled output z, when it is in the steady state, i.e., when the augmented state variable is inside the set of attraction $\mathcal{X}$. Notice that, in our context, the role of the switched filter is restricted to providing information for the switching rule and, therefore, its steady state behavior is not of interest. For this reason, a measure of $\mathcal{X}$ has to be defined depending only on the system state $\xi \in \mathbb{R}^{n_x}$. Before presenting the results concerning the output feedback control design, we firstly provide alternative state feedback design conditions, which imply the ones of [20] and are more amenable for output feedback generalization.

III. Main results

A. State feedback control design

Let us consider the general quadratic Lyapunov function

$$v(\xi) = \begin{bmatrix} \xi \\ 1 \end{bmatrix}' \begin{bmatrix} P & h \\ h' & h'P^{-1}h \end{bmatrix} \begin{bmatrix} \xi \\ 1 \end{bmatrix} \tag{7}$$

with $h \in \mathbb{R}^{n_x}$ and $0 < P \in \mathbb{R}^{n_x \times n_x}$, written alternatively as $v(\xi) = (\xi - \xi_c)'P(\xi - \xi_c)$ with $\xi_c = -P^{-1}h$. Clearly, this is a convex function such that $v(\xi) > 0$ for all $\xi \neq \xi_c$ and $v(\xi_c) = 0$. Reference [22] considers a Lyapunov function with $\xi_c = 0$, while in [20] the center ξ_c is determined from the equilibrium solution to a minimax problem. The next theorem presents the conditions for the state feedback case.

Theorem 1: Consider system (3) with $C_i = I,\ \forall i \in \mathbb{K}$, and let the equilibrium point $x_e \in X_e$ be given with its associated vector $\lambda \in \Lambda$. If there exist symmetric matrices W, P, Q_i, vectors $h, g, c_i \in \mathbb{R}^{n_x}$ and scalars ρ_i solution to the convex optimization problem

$$\inf_{W,P,Q_i,h,g,c_i,\rho_i} -\ln(\det(W)) \tag{8}$$

subject to the LMIs

$$\begin{bmatrix} P - Q_i & \bullet & \bullet \\ c_i' + h'(I - A_i) & \rho_i - \ell_i' h - h'\ell_i & \bullet \\ PA_i & P\ell_i & P \end{bmatrix} > 0,\ i \in \mathbb{K} \tag{9}$$

$$\begin{bmatrix} Q_\lambda & \bullet & \bullet \\ -c_\lambda' & 1 - \rho_\lambda & \bullet \\ W & g & W \end{bmatrix} > 0 \tag{10}$$

then, the state feedback switching function

$$\sigma(\xi) = \arg\min_{i \in \mathbb{K}} -\xi' Q_i \xi + 2c_i'\xi + \rho_i \tag{11}$$

assures that the equilibrium point $\xi = 0$ is globally practically stable and that

$$\mathcal{X} = \{\xi \in \mathbb{R}^{n_x}\ :\ (\xi - \mu)'W(\xi - \mu) \le 1\} \tag{12}$$

where $\mu = -W^{-1}g$, is an ellipsoidal set of attraction of minimum volume.

Proof: Define the difference operator of (7) as $\Delta v(\xi[k]) = v(\xi[k+1]) - v(\xi[k])$ and assume that the inequalities (9)-(10) are verified. The time dependency is omitted from this point on to easy the presentation. Evaluating this operator for an arbitrary trajectory of (3), we have

$$\begin{aligned} \Delta v(\xi) &= v(A_\sigma \xi + \ell_\sigma) - v(\xi) \\ &= \begin{bmatrix} \xi \\ 1 \end{bmatrix}' \begin{bmatrix} A_\sigma' P A_\sigma - P & \bullet \\ \ell_\sigma' P A_\sigma + h'(A_\sigma - I) & \ell_\sigma' P \ell_\sigma + 2\ell_\sigma' h \end{bmatrix} \begin{bmatrix} \xi \\ 1 \end{bmatrix} \\ &< \begin{bmatrix} \xi \\ 1 \end{bmatrix}' \begin{bmatrix} -Q_\sigma & \bullet \\ c_\sigma' & \rho_\sigma \end{bmatrix} \begin{bmatrix} \xi \\ 1 \end{bmatrix} \end{aligned} \tag{13}$$

where this inequality is verified from (9) after applying the Schur Complement Lemma with respect to the third row and column. Now, adopting the switching function (11), we obtain

$$\begin{aligned} \Delta v(\xi) &< \min_{i \in \mathbb{K}} \begin{bmatrix} \xi \\ 1 \end{bmatrix}' \begin{bmatrix} -Q_i & \bullet \\ c_i' & \rho_i \end{bmatrix} \begin{bmatrix} \xi \\ 1 \end{bmatrix} \\ &= \min_{\lambda \in \Lambda} \begin{bmatrix} \xi \\ 1 \end{bmatrix}' \begin{bmatrix} -Q_\lambda & \bullet \\ c_\lambda' & \rho_\lambda \end{bmatrix} \begin{bmatrix} \xi \\ 1 \end{bmatrix} \\ &\le \begin{bmatrix} \xi \\ 1 \end{bmatrix}' \begin{bmatrix} -Q_\lambda & \bullet \\ c_\lambda' & \rho_\lambda \end{bmatrix} \begin{bmatrix} \xi \\ 1 \end{bmatrix} \end{aligned} \tag{14}$$

which holds for all $\xi \in \mathbb{R}^{n_x}$. Defining the set $\mathcal{X}_s$ as being

$$\mathcal{X}_s = \{\xi \in \mathbb{R}^{n_x} : (\xi - \mu_s)'Q_\lambda(\xi - \mu_s) \le c_\lambda' Q_\lambda^{-1} c_\lambda + \rho_\lambda\} \tag{15}$$

with $\mu_s = Q_\lambda^{-1} c_\lambda$, and considering the validity of (14), we have the following two properties. Firstly, $\xi = 0 \in \mathcal{X}_s$ because from (9) we have that $\rho_i > 2\ell_i' h$, which implies that $\rho_\lambda > 0$ since $\ell_\lambda = 0$. The second is that $\Delta v(\xi) < 0,\ \forall \xi \notin \mathcal{X}_s$, indicating that $\mathcal{X}_s$ is a set of attraction for the system. See [20] for a formal definition of the set of attraction. Moreover, notice by $\mathcal{S}$-procedure that the set $\mathcal{X}$, defined in (12), contains the ellipsoid $\mathcal{X}_s$ whenever the inequality

$$\begin{bmatrix} W & \bullet \\ -\mu' W & \mu' W \mu - 1 \end{bmatrix} < \beta \begin{bmatrix} Q_\lambda & \bullet \\ -c_\lambda' & -\rho_\lambda \end{bmatrix} \tag{16}$$

is satisfied for $\beta > 0$, see [26] for details. In other words, multiplying both sides of (16) to the left by $[\xi' \ \ 1]$ and to the right by its transpose, this inequality assures that $(\xi - \mu)'W(\xi - \mu) \le 1$ whenever $\xi \in \mathcal{X}_s$ and, as a consequence, $\mathcal{X}$ is a set of attraction, since $\mathcal{X} \supset \mathcal{X}_s$. By applying the Schur Complement suitably in (16) we can rewrite it as

$$\begin{bmatrix} \beta Q_\lambda & \bullet & \bullet \\ -\beta c_\lambda' & 1 - \beta\rho_\lambda & \bullet \\ W & g & W \end{bmatrix} > 0 \tag{17}$$

Now, notice that (9) can be multiplied by the scalar β, without loss of generality. After redefining, with some abuse of notation, $(Q_i, c_i, \rho_i, P, h) \to (\beta Q_i, \beta c_i, \beta\rho_i, \beta P, \beta h)$, the resultant inequality together with (17) are exactly the inequalities (9)-(10), assuring that $\mathcal{X}$ is an ellipsoidal set of attraction. The objective function (8) is responsible for minimizing the volume of (12). The proof is concluded. □

At this point, it is important to show the relationship between this result and that of Theorem 1 of [20]. Indeed, that theorem establishes that, for a given $x_e \in X_e$ and its associated vector $\lambda \in \Lambda$, the set of attraction $\mathcal{X} = \{\xi \in \mathbb{R}^{n_x} : \xi' W \xi \le 1\}$ of minimum volume is obtained as the solution to the convex optimization problem

$$\inf_{P>0, W>0} -\ln(\det(W)) \tag{18}$$

subject to

$$\sum_{i \in \mathbb{K}} \lambda_i A_i' P A_i - P < -W, \quad \sum_{i \in \mathbb{K}} \ell_i' P \ell_i < 1 \tag{19}$$

where the switching function $\sigma(\xi) = \arg\min_{i \in \mathbb{K}} v(A_i \xi + \ell_i)$ is dependent on $h = (I - A_\lambda')^{-1}(\sum_{i \in \mathbb{K}} \lambda_i A_i' P \ell_i)$, which is the equilibrium solution to the minimax problem

$$\min_{h \in \mathbb{R}^{n_x}} \max_{\xi \in \mathbb{R}^{n_x}} f(\xi, h) \tag{20}$$

with $f(\xi, h)$ being the upper bound on $\Delta v(\xi) \le f(\xi, h)$. In our context, notice that (9), after applying the Schur Complement with respect to the last row and column, can be rewritten as

$$\begin{bmatrix} A_i' P A_i - P & \bullet \\ \ell_i' P A_i - h'(I - A_i) & \ell_i' P \ell_i + 2\ell_i' h \end{bmatrix} < \begin{bmatrix} -Q_i & \bullet \\ c_i' & \rho_i \end{bmatrix} \tag{21}$$

We can observe that, without loss of generality, the right hand side of this inequality can be taken arbitrarily close to the left hand side and, therefore, we obtain the same switching function of [20]. Moreover, imposing $g = 0$, inequality (21) together with (10), after performing the Schur Complement Lemma with respect to the last row and column, provide

$$\begin{bmatrix} \sum_{i \in \mathbb{K}} \lambda_i A_i' P A_i - P & \bullet \\ \sum_{i \in \mathbb{K}} \lambda_i \ell_i' P A_i - h'(I - A_\lambda) & \sum_{i \in \mathbb{K}} \lambda_i \ell_i' P \ell_i \end{bmatrix} < \begin{bmatrix} -W & \bullet \\ 0 & 1 \end{bmatrix} \tag{22}$$

where it has been used the fact that $\ell_\lambda = 0$ since $x_e \in X_e$. Notice that $h = (I - A'_\lambda)^{-1}(\sum_{i\in\mathbb{K}} \lambda_i A'_i P \ell_i)$ is the best choice, since this vector makes null the off main diagonal element, implying that (22) is equivalent to $\sum_{i\in\mathbb{K}} \lambda_i A'_i P A_i - P < -W$ and $\sum_{i\in\mathbb{K}} \lambda_i \ell'_i P \ell_i < 1$. Hence, the resultant inequalities become exactly (19), putting in evidence that the result of Theorem 1 implies the one of [20]. It is important to remark that the equivalence of these practical stability conditions occurs when $\mu = 0$ is enforced. Furthermore, as it will be clear later, the generalization of these conditions to cope with the output feedback control design requires to impose structures to some matrix variables in order to make the switching rule independent of the system state, as well as the objective function independent of the filter state. For this purpose, Theorem 1 of the present paper is more amenable for output feedback generalization.

Two main properties of Theorem 1 worth to be mentioned. The first one regards stability. Notice that the necessary and sufficient condition for feasibility is $\sum_{i\in\mathbb{K}} \lambda_i A'_i P A_i - P < 0$ and, therefore, nothing is required from the matrices $A_i,\ \forall i \in \mathbb{K}$, considered separately. The second property refers to the generalization to cope with sampled-data control of continuous-time switched affine systems. Indeed, the proposed conditions become those of [3] whenever the sampling period T tends to zero, see Section IV of [20] for details. Finally, it is important to verify whether the center of the Lyapunov function $\xi_c = -P^{-1}h$ and the equilibrium point $\xi = 0$ belong to the set of attraction $\mathcal{X}$. To demonstrate that $\xi_c \in \mathcal{X}$, notice that (14) evaluated for ξ_c provides

$$\begin{bmatrix}\xi_c\\1\end{bmatrix}'\begin{bmatrix}-Q_\lambda & \bullet\\ c'_\lambda & \rho_\lambda\end{bmatrix}\begin{bmatrix}\xi_c\\1\end{bmatrix} > v(A_\sigma\xi_c + \ell_\sigma) \geq 0 \tag{23}$$

because $v(\xi_c) = 0$. This inequality, together with (10), after applying the Schur Complement with respect to the last row and column, produces

$$\begin{bmatrix}\xi_c\\1\end{bmatrix}'\begin{bmatrix}W & \bullet\\ -\mu'W & \mu'W\mu - 1\end{bmatrix}\begin{bmatrix}\xi_c\\1\end{bmatrix} < \begin{bmatrix}\xi_c\\1\end{bmatrix}'\begin{bmatrix}Q_\lambda & \bullet\\ -c'_\lambda & -\rho_\lambda\end{bmatrix}\begin{bmatrix}\xi_c\\1\end{bmatrix} < 0 \tag{24}$$

which guarantees that $\xi_c \in \mathcal{X}$. Now, we have that $0 \in \mathcal{X}$ because $0 \in \mathcal{X}_s$, as proved in Theorem 1, and $\mathcal{X}_s \subset \mathcal{X}$.

B. Output feedback control design

At this point, our main goal is to generalize the conditions of Theorem 1 to cope with the augmented system (5) taking into account the structured matrices

$$\tilde{Q}_i = \begin{bmatrix}Q & \bar{Q}\\ \bar{Q}' & \hat{Q}_i\end{bmatrix},\ \tilde{c}_i = \begin{bmatrix}c\\ \hat{c}_i\end{bmatrix},\ \tilde{\rho}_i = \rho_i \tag{25}$$

This structure is important to make the switching rule dependent only on the measured output through the filter state variable $\hat{\xi}$, that is,

$$\begin{aligned}\sigma(\hat{\xi}) &= \arg\min_{i\in\mathbb{K}} -\tilde{\xi}'\tilde{Q}_i\tilde{\xi} + 2\tilde{c}'_i\tilde{\xi} + \tilde{\rho}_i\\ &= \arg\min_{i\in\mathbb{K}} -\hat{\xi}'\hat{Q}_i\hat{\xi} + 2\hat{c}'_i\hat{\xi} + \rho_i\end{aligned} \tag{26}$$

Although the filter is essential for the switching function implementation, the optimization of the set of attraction must be focused only on the subspace generated by the system state variable $\xi \in \mathbb{R}^{n_x}$.

In order to present a set of attraction independent of $\hat{\xi}$, let us define the degenerated ellipsoid

$$\tilde{\mathcal{X}} = \{\tilde{\xi} \in \mathbb{R}^{2n_x} : (\tilde{\xi} - \tilde{\mu})'\tilde{W}(\tilde{\xi} - \tilde{\mu}) \leq 1\} \tag{27}$$

with structured matrices

$$\tilde{W} = \begin{bmatrix}W & 0\\ 0 & 0\end{bmatrix},\ \tilde{\mu} = \begin{bmatrix}\mu\\0\end{bmatrix} \tag{28}$$

where $W > 0$. Notice that the ellipsoid $\mathcal{X}$ given by

$$\mathcal{X} = \{\xi \in \mathbb{R}^{n_x} : (\xi - \mu)'W(\xi - \mu) \leq 1\} \tag{29}$$

can be interpreted as the projection of a general ellipsoid $\tilde{\mathcal{X}}$ on the subspace generated by $\xi \in \mathbb{R}^{n_x}$.

The next theorem presents the control design conditions of an output-dependent switching function, assuring practical stability through the volume minimization of the set of attraction given by (29).

Theorem 2: Consider the system (3) and let the equilibrium point $x_e \in X_e$ be given with its associated vector $\lambda \in \Lambda$. If there exist symmetric matrices Z, Q, $\hat{Q}_i$, Y, W, matrices J, L_i, M_i, vectors ν_i, c, $\hat{c}_i$, h, g, and scalars ρ_i, solution to the convex optimization problem

$$\inf_{\mathcal{F}} -\ln(\det(W)) \tag{30}$$

with $\mathcal{F}$ being the set of feasible decision variables satisfying

$$\begin{bmatrix}Z-Q-J-J'-\hat{Q}_i & \bullet & \bullet & \bullet & \bullet\\ Z-Q-J & Y-Q & \bullet & \bullet & \bullet\\ c'+\hat{c}'_i+h'(I-A_i) & c'+h'(I-A_i) & \rho_i-2\ell'_i h & \bullet & \bullet\\ ZA_i & ZA_i & Z\ell_i & Z & \bullet\\ YA_i+L_iC_i+M_i & YA_i+L_iC_i & Y\ell_i+\nu_i & Z & Y\end{bmatrix} > 0 \tag{31}$$

for all $i \in \mathbb{K}$ and

$$\begin{bmatrix}Q+J+J'+\hat{Q}_\lambda & \bullet & \bullet & \bullet\\ Q+J & Q & \bullet & \bullet\\ -c'-\hat{c}'_\lambda & -c' & 1-\rho_\lambda & \bullet\\ W & W & g & W\end{bmatrix} > 0 \tag{32}$$

then, the output feedback switching function (26) and the switched filter (4) given by matrices

$$\hat{A}_i = (Z-Y)^{-1}M_i, \hat{B}_i = (Z-Y)^{-1}L_i, \hat{\ell}_i = (Z-Y)^{-1}\nu_i \tag{33}$$

assure that the state trajectories $\xi[k],\ \forall k \in \mathbb{N}$ of (3) converge to the ellipsoidal set of attraction

$$\mathcal{X}_* = \{\xi \in \mathbb{R}^{n_x} : (\xi - \mu)'W(\xi - \mu) \leq 1\} \tag{34}$$

with $\mu = -W^{-1}g$, of minimum volume.

Proof: The proof consists in demonstrating that the LMIs (31) and (32) assure the validity of (9) and (10), respectively, whenever the augmented matrices (6) and the structured ones (25) and (28) are taken into account with $\tilde{W}$ of the form $\tilde{W} = \mathrm{diag}(W, \epsilon I)$ where $\epsilon > 0$ is arbitrarily small. Firstly, let us define the partitioned matrices

$$\tilde{P} = \begin{bmatrix}Y & V\\ V' & \hat{Y}\end{bmatrix}, \tilde{S} = \tilde{P}^{-1} = \begin{bmatrix}X & U\\ U' & \hat{X}\end{bmatrix}, \tilde{h} = \begin{bmatrix}h\\0\end{bmatrix}, \tilde{\Gamma} = \begin{bmatrix}X & I\\ U' & 0\end{bmatrix} \tag{35}$$

Notice that inequality (9), multiplied to the left by $\mathrm{diag}(\tilde{\Gamma}', I, \tilde{\Gamma}')$ and to the right by its transpose, yields

$$\begin{bmatrix} \tilde{\Gamma}'\tilde{P}\tilde{\Gamma} - \tilde{\Gamma}'\tilde{Q}_i\tilde{\Gamma} & \bullet & \bullet \\ \tilde{c}_i'\tilde{\Gamma} + \tilde{h}'(I-\tilde{A}_i)\tilde{\Gamma} & \tilde{\rho}_i - \tilde{\ell}_i'\tilde{h} - \tilde{h}'\tilde{\ell}_i & \bullet \\ \tilde{\Gamma}'\tilde{P}\tilde{A}_i\tilde{\Gamma} & \tilde{\Gamma}'\tilde{P}\tilde{\ell}_i & \tilde{\Gamma}'\tilde{P}\tilde{\Gamma} \end{bmatrix} > 0, i \in \mathbb{K} \quad (36)$$

while inequality (10) multiplied to the left by $\mathrm{diag}(\tilde{\Gamma}', I, \tilde{W}^{-1})$ and to the right by the transpose produces

$$\begin{bmatrix} \tilde{\Gamma}'\tilde{Q}_\lambda\tilde{\Gamma} & \bullet & \bullet \\ -\tilde{c}_\lambda'\tilde{\Gamma} & 1-\tilde{\rho}_\lambda & \bullet \\ \tilde{\Gamma} & -\tilde{\mu} & \tilde{W}^{-1} \end{bmatrix} > 0 \quad (37)$$

where the last row and column are eliminated by making $\epsilon > 0$ arbitrarily small. From the fact that $\tilde{P}\tilde{S} = I$, we obtain the identities $YX + VU' = I$, $YU + V\hat{X} = 0$, $V'X + \hat{Y}U' = 0$ and $V'U + \hat{Y}\hat{X} = I$ which allows us to calculate

$$\tilde{\Gamma}'\tilde{P}\tilde{\Gamma} = \begin{bmatrix} X & \bullet \\ I & Y \end{bmatrix}, \; \tilde{\Gamma}'\tilde{P}\tilde{A}_i\tilde{\Gamma} = \begin{bmatrix} A_iX & A_i \\ \Phi_i & YA_i + V\hat{B}_iC_i \end{bmatrix} \quad (38)$$

$$\tilde{\Gamma}'\tilde{P}\tilde{\ell}_i = \begin{bmatrix} \ell_i \\ Y\ell_i + V\hat{\ell}_i \end{bmatrix}, \tilde{\Gamma}'\tilde{h} = \begin{bmatrix} Xh \\ h \end{bmatrix}, \tilde{\Gamma}'\tilde{c}_i = \begin{bmatrix} Xc + U\hat{c}_i \\ c \end{bmatrix} \quad (39)$$

$$\Gamma'\tilde{A}_i'\tilde{h} = \begin{bmatrix} XA_i'h \\ A_i'h \end{bmatrix}, \; \tilde{\Gamma}'\tilde{Q}_i\tilde{\Gamma} = \begin{bmatrix} \Psi_i & \bullet \\ QX + \bar{Q}U' & Q \end{bmatrix} \quad (40)$$

with $\Phi_i = YA_iX + V\hat{B}_iC_iX + V\hat{A}_iU'$ and $\Psi_i = XQX + X\bar{Q}U' + U\bar{Q}'X + U\hat{Q}_iU'$. Replacing these identities into (36) and (37) and multiplying both sides of the first one by $\mathrm{diag}(Z, I, I, Z, I)$ and the second by $\mathrm{diag}(Z, I, I, W)$, with $Z = X^{-1}$, we obtain

$$\begin{bmatrix} Z - Z\Psi_iZ & \bullet & \bullet & \bullet & \bullet \\ Z - Q - \bar{Q}U'Z & Y - Q & \bullet & \bullet & \bullet \\ \Pi_i & c' + h'(I - A_i) & \rho_i - 2\ell_i'h & \bullet & \bullet \\ ZA_i & ZA_i & Z\ell_i & Z & \bullet \\ \Phi_iZ & YA_i + V\hat{B}_iC_i & Y\ell_i + V\hat{\ell}_i & Z & Y \end{bmatrix} > 0 \quad (41)$$

$$\begin{bmatrix} Z\Psi_\lambda Z & \bullet & \bullet & \bullet \\ Q + \bar{Q}U'Z & Q & \bullet & \bullet \\ -c' - \hat{c}_\lambda'U'Z & -c' & 1 - \rho_\lambda & \bullet \\ W & W & -W\mu & W \end{bmatrix} > 0 \quad (42)$$

with $\Pi_i = c' + \hat{c}_i'U'Z + h'(I - A_i)$, respectively. By replacing the variables $R_i = ZU\hat{Q}_iU'Z$, $J = \bar{Q}U'Z$, $M_i = V\hat{A}_iU'Z$, $L_i = V\hat{B}_i$, $\nu_i = V\hat{\ell}_i$, $d_i' = \hat{c}_i'U'Z$, notice that both (41) and (42) become linear and independent of U and V. This shows that, without loss of generality, one of these matrix variables can be chosen arbitrarily such that $\det(V) \neq 0$ or $\det(U) \neq 0$, while the other has to be determined from the identity $YX + VU' = I$, which is one of the conditions that assure $\tilde{P}\tilde{S} = I$. Therefore, choosing matrix $U = X$ we have $V = Z - Y$, which provides the filter matrices (33). Moreover, inequalities (41) and (42) become (31) and (32), which assure the validity of (9) and (10) applied to the augmented system (5) and considering variables with the structure specified in (25), (28) and $\tilde{h} = [h' \; 0]'$ in (35). It is worth remembering that the structure of (25) is essential to make the switching rule dependent only on the filter state variable as demonstrated in (26). Finally, notice that $\tilde{\xi} = \tilde{\xi}_c \in \mathcal{X}$ and $\tilde{\xi} = 0 \in \mathcal{X}$ by the same arguments drawn in the state feedback case. The proof is concluded. □

This result generalizes Theorem 1 to cope with dynamic output feedback control design. The objective function is a nonlinear convex function and the design conditions are expressed in terms of LMIs, which is a problem well adapted to be solved by the celebrated Frank-Wolfe algorithm, as in [20].

An interesting aspect of this result is that the optimal set of attraction obtained from Theorem 2 is exactly the same as the one obtained from Theorem 1 whenever the pairs (A_i, C_i), $i \in \mathbb{K}$, are quadratically detectable. This fact is formally stated in the next theorem.

Theorem 3: Consider that the optimal solution to the optimization problem (8)-(10) occurs for $(W^*, P^*, Q_i^*, h^*, g^*, c_i^*, \rho_i^*)$. If the pairs (A_i, C_i), $i \in \mathbb{K}$, are quadratically detectable, that is, for all $i \in \mathbb{K}$, there exist $R > 0$ and $\hat{B}_i$ such that

$$(A_i - \hat{B}_iC_i)'R(A_i - \hat{B}_iC_i) - R < 0 \quad (43)$$

then the same matrix $W^* > 0$ forms the optimal solution to (30)-(32). Moreover, the switching function (26) can be defined with the triple (Q_i^*, c_i^*, ρ_i^*) which yields

$$\sigma(\hat{\xi}) = \arg\min_{i \in \mathbb{K}} -\hat{\xi}'Q_i^*\hat{\xi} + 2c_i^{*\prime}\hat{\xi} + \rho_i^* \quad (44)$$

and the associated switched filter (4) admits the observer form

$$\hat{A}_i = A_i - \hat{B}_iC_i, \; \hat{\ell}_i = \ell_i \quad (45)$$

Proof: Take R and $\hat{B}_i$, $i \in \mathbb{K}$ satisfying (43) and the solution to (8)-(10) formed by $(W^*, P^*, Q_i^*, h^*, g^*, c_i^*, \rho_i^*)$. The proof consists in showing that a feasible solution to (30)-(32) is formed by the choices $W = W^*$, $Z = P^*$, $h = h^*$, $Q = W^* + \epsilon I$, $J = -\epsilon I$, $\hat{Q}_i = Q_i^* - W^* + \epsilon I$, $c = -g^*$, $g = g^*$, $\hat{c}_i = g^* + c_i^*$, $\rho_i = \rho_i^*$, $M_i = -YA_i - L_iC_i$, $\nu_i = -Y\ell_i$, $L_i = -Y\hat{B}_i$ and $Y = \alpha R$ where the positive scalars $\epsilon, \alpha \in \mathbb{R}$ are such that $\epsilon \to 0$ and $\alpha \to \infty$. To show that (31) holds, let us permutate its second and fourth rows and columns and subsequently its second and third rows and columns to obtain an equivalent LMI that can be split into two inequalities by using the Schur Complement Lemma. These inequalities are $\mathcal{W}_i > 0$ and $\mathcal{Y}_i - \mathcal{U}_i'\mathcal{W}_i^{-1}\mathcal{U}_i > 0$ for all $i \in \mathbb{K}$ where

$$\mathcal{Y}_i = \begin{bmatrix} Y - Q & \bullet \\ YA_i + L_iC_i & Y \end{bmatrix}, \mathcal{W}_i = \begin{bmatrix} Z - Q - J - J' - \hat{Q}_i & \bullet & \bullet \\ c' + \hat{c}_i' + h'(I - A_i) & \rho_i - 2\ell_i'h & \bullet \\ ZA_i & Z\ell_i & Z \end{bmatrix}$$

$$\mathcal{U}_i = \begin{bmatrix} Z - Q - J & c + (I + A_i)'h & A_i'Z \\ YA_i + L_iC_i + M_i & Y\ell_i + \nu_i & Z \end{bmatrix}$$

For the chosen variables, the first inequality becomes (9), which is assumed to be true. Moreover, after replacing the chosen variables, notice that $Y = \alpha R$ appears only in $\mathcal{Y}_i$ and, as a consequence, the second inequality always holds due to quadratic detectability (43), obtained from $\mathcal{Y}_i > 0$, and the fact that α is large enough. To show that (32) also holds, let us now multiply it to the left by Θ and to the right by Θ' with

$$\Theta = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ 0 & I & 0 & -I \end{bmatrix} \quad (46)$$

yielding an equivalent inequality that holds due to the validity of (10). Finally, the switching function (44) comes directly from (26) using the chosen variables and from (33) we obtain that $\hat{A}_i = A_i - \hat{B}_i C_i$ and $\hat{\ell}_i = \ell_i$ because $Z - Y = P^* - \alpha R \approx -\alpha R$ since $\alpha \to \infty$, demonstrating that a valid optimal solution produces a filter with the observer form. □

It is also important to remark that quadratic detectability is not only a sufficient condition but also necessary for the feasibility of (30)-(32) given that (8)-(10) is feasible. This can be straightforwardly verified from $\mathcal{W}_i > 0$ that imposes $\mathcal{Y}_i > 0$ for all $i \in \mathbb{K}$. It is noteworthy that it is the only additional requirement with respect to Theorem 1. The following example highlights this point.

Example 1: Consider the switched affine system (1) with

$$A_1 = \begin{bmatrix} -3.8 & -5 \\ 2 & 2.7 \end{bmatrix},\ A_2 = \begin{bmatrix} -2.6 & -5 \\ 2 & 3.9 \end{bmatrix}, \tag{47}$$

$b_1 = [0\ 0.5]'$, $b_2 = [0.4\ -0.5]$, $C_1 = [0.4\ 0.5]$ and $C_2 = [0.1\ 0.2]$. Our control goal is to design an output-dependent switching strategy to globally govern the state trajectories towards $x_e = [-1.4274\ 1.2522]' \in X_e$ associated with $\lambda = [0.55\ 0.45]' \in \Lambda$. Notice that both subsystems, considered separately, are unstable and not observable. In spite of that, Theorem 2 is capable of assuring global practical stability of x_e. Indeed, solving the correspondent optimization problem provided us with a set of attraction (34) where

$$W = \begin{bmatrix} 2.2412 & 4.3060 \\ 4.3060 & 8.4198 \end{bmatrix},\ g = 0 \tag{48}$$

It coincides with the one obtained by solving (8)-(10), which verifies the statements of Theorem 3. Simulations to validate the switching rule (44) and the observer (45) were carried out but omitted due to limited space.

An alternative objective function is presented in the next corollary that can be very useful in practical applications, when the designer is interested in a specific controlled output instead of the full state variable $\xi \in \mathbb{R}^{n_x}$.

Corollary 1: Consider the system (3) and let the equilibrium point $x_e \in X_e$ be given with its associated $\lambda \in \Lambda$. If there exist symmetric matrices Z, Q, $\hat{Q}_i$, Y, matrices J, L_i, M_i, vectors ν_i, c, $\hat{c}_i$, h, and scalars ρ_i, γ solution to the problem

$$\sup_{\mathcal{F}} \gamma \tag{49}$$

with $\mathcal{F}$ being the set of feasible decision variables satisfying (31) for all $i \in \mathbb{K}$ and

$$\begin{bmatrix} Q + J + J' + \hat{Q}_\lambda - \gamma E'E & \bullet & \bullet \\ Q + J - \gamma E'E & Q - \gamma E'E & \bullet \\ -c' - \hat{c}_\lambda' & -c' & 1 - \rho_\lambda \end{bmatrix} \geq 0 \tag{50}$$

then, the output feedback switching function (26) and the filter (4) given by matrices (33) assure that the state trajectories ξ of (3) converge to the ball

$$\mathcal{B}_* = \{ z \in \mathbb{R}^{n_z}\ :\ z'z \leq \gamma^{-1} \} \tag{51}$$

Proof: The proof comes from the validity of Theorem 2. Indeed, performing the Schur Complement Lemma to (32) with respect to the last row and column, we can show that (50) is equivalent to (32) with $W = \gamma E'E + \epsilon I$, $\epsilon > 0$ arbitrarily small, and $g = 0$. With this choice, the set of attraction (34) becomes (51). □

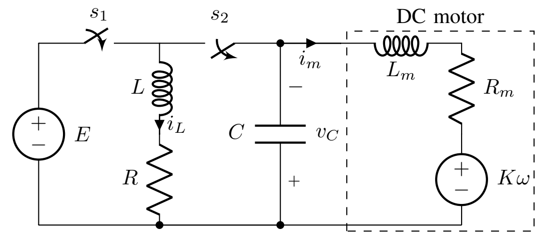

Fig. 1. Schematic of a Buck-boost DC-DC converter feeding a DC motor.

In certain practical applications, this solution can be of greater interest than the volume minimization, mainly when the idea is to deal with a specific controlled output.

IV. Practical application example

Within this section, simulation results demonstrating the practical interest on the developed theory are presented. This example deals with the velocity control of a DC motor fed by a buck-boost power converter. It has already been considered in [4] to illustrate the state feedback control design exclusively. The electric schematic model is depicted in Figure 1. Taking into account that the switches s_1 and s_2 are alternately commanded, such that $\sigma(t) = 1$ when s_1 is closed and $\sigma(t) = 2$, otherwise, and that $\sigma(t) = i$, for $t \in [t_k, t_{k+1})$ with $t_{k+1} - t_k = T$, $\forall k \in \mathbb{N}$, this system can be modeled as the discrete-time switched system (1) with matrices $A_i = e^{F_i T}$, $b_i = \int_0^T e^{F_i \tau} d\tau\ g_i$, $i \in \mathbb{K}$, being

$$F_1 = \begin{bmatrix} \frac{-(R_L+R_s)}{L} & 0 & 0 \\ 0 & -1/(R_m C) & -K_e/(R_m C) \\ 0 & -K_m/(J R_m) & \frac{-T_v R_m - K_m K_e}{J R_m} \end{bmatrix},$$

$$F_2 = \begin{bmatrix} -R_L/L & -1/L & 0 \\ 1/C & -1/(R_m C) & -K_e/(R_m C) \\ 0 & -K_m/(J R_m) & \frac{-T_v R_m - K_m K_e}{J R_m} \end{bmatrix},$$

$$g_1 = \begin{bmatrix} E/L \\ 0 \\ T_d/J \end{bmatrix},\quad g_2 = \begin{bmatrix} 0 \\ 0 \\ T_d/J \end{bmatrix}$$

where $x[k] = [i_L[k]\ v_C[k]\ \omega[k]]'$. The electrical and mechanical parameters are $C = 2.2 \times 10^{-3}$ [F], $E = 10.75$ [V], $K_e = 0.051$ [V.s/rad], $K_m = 0.051$ [N.m/A], $L = 8.5 \times 10^{-3}$ [H], $R_L = 1.129$ [Ω], $R_m = 10.207$ [Ω], $R_s = 0.450$ [Ω], $J = 3.792 \times 10^{-3}$ [kg.m^2], $T_d = 6.195 \times 10^{-3}$ [N.m] and $T_v = 76.220 \times 10^{-6}$ [N.m.s/rad] and it has been considered a sampling period of $T = 10^{-3}$ [s]. In contrast to reference [4], only the rotational velocity $\omega[k]$ is measured, i.e., $C_1 = C_2 = [0\quad 0\quad 1]$ in (1). No information about the electrical variables $i_L[k]$ and $v_C[k]$ is provided to the controller. In this situation, our goal is to bring $\omega[k]$ as close as possible to a desired velocity ω_e and, for this purpose, we have considered two approaches, i.e., the volume minimization presented in Theorem 2 and the output optimization of Corollary 1 for $E = [0.5\quad 0\quad 1]$.

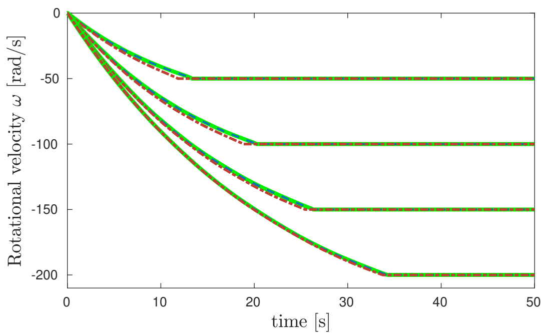

Fig. 2. Rotational velocity trajectory for several ω_e obtained from the solution to Theorem 1 (dashed), 2 (solid) and Corollary 1 (dot-dashed).

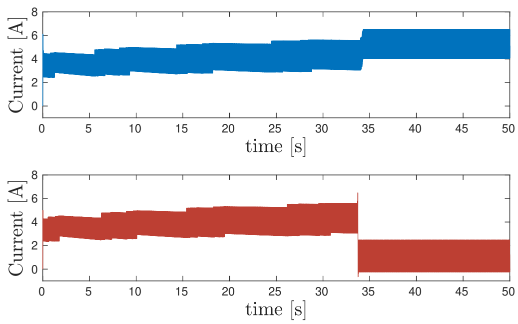

Fig. 3. Currents $i_L[k]$ for $\omega_e = -200.00$ [rad/s] obtained from Theorem 2 (top) and Corollary 1 (bottom).

The reference velocities are $\omega_e \in \{-50, -100, -150, -200\}$ [rad/s]. Solving the conditions of Theorem 2 and Corollary 1 we have implemented the filter (4), evolving from $\hat{\xi}_0 = 0$, and the switching function (11), for $x[0] = 0$, obtaining the trajectories presented in Figure 2. For the sake of comparison, we have presented in the same figure, the trajectories obtained from Theorem 1. The correspondent optimal values for the volume $vol(\mathcal{X})$ and the square radius γ^{-1} are given in Table I together with the first associated component of λ, used in the optimization procedure. Notice that the volumes provided by Theorems 1 and 2 are practically coincident as expected from the result of Theorem 3. The associated current trajectories are presented in Figure 3 where it is clear that the steady state current is closer to $i_{Le} = 2.54$ [A] for the approach given in Corollary 1. Finally, for $\omega_e = -200.00$ [rad/s], Figure 4 depicts the trajectories $\xi[k]$ at the top and a zoomed image concerning the steady state behavior at the bottom with the sets $\mathcal{X}_*$ and $\mathcal{B}_*$, defined in (34) and (51), respectively.

TABLE I
OPTIMAL VALUES FOR THEOREMS 1 (T1), 2 (T2) AND COROLLARY 1 (C1) WITH CORRESPONDENT ω_e AND λ.

		T1	T2	C1
ω_e	λ_1	$vol(\mathcal{X})$	$vol(\mathcal{X})$	γ^{-1}
-50.00	0.3552	51.4150	51.4601	1.1879
-100.00	0.5089	101.6993	101.8848	2.1517
-150.00	0.6257	130.5696	130.5829	2.8817
-200.00	0.7291	108.9545	109.0687	2.8663

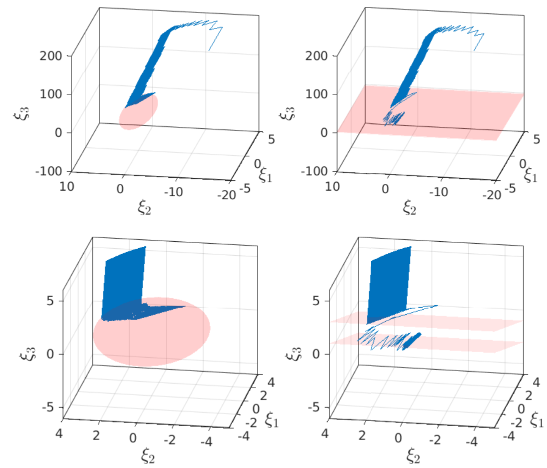

Fig. 4. Trajectories $\xi[k]$ for $\omega_e = -200.00$ [rad/s] from Theorem 2 (left) and Corollary 1 (right), with the correspondent sets $\mathcal{X}_*$ and $\mathcal{B}_*$, respectively.

V. CONCLUSION

In this paper, a design procedure to an output-dependent switching function for discrete-time switched affine systems has been presented, guaranteeing practical stability of a desired equilibrium point. To the best of the authors' knowledge, this is the first result dealing with the output feedback control problem for this class of switched systems. The presented switching function along with the switched affine filter have guaranteed global convergence to a set of attraction and two different objective functions have been taken into account. A practical application consisting of the velocity control for a DC motor driven by a buck-boost converter has been used to validate the proposed methodology, showing its efficiency and usefulness.

REFERENCES

[1] R. Cardim, M. C. M. Teixeira, E. Assuncao, and M. R. Covacic, "Variable-structure control design of switched systems with an application to a DC–DC power converter," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 9, pp. 3505–3513, 2009.

[2] A. Sferlazza, C. Albea-Sanchez, L. Martínez-Salamero, , G. Garcia, and C. Alonso, "Min-type control strategy of a DC-DC synchronous boost converter," *IEEE Transactions on Industrial Electronics*, vol. 67, no. 4, pp. 3167–3179, 2019.

[3] G. S. Deaecto, J. C. Geromel, F. S. Garcia, and J. A. Pomilio, "Switched affine systems control design with application to DC–DC converters," *IET Control Theory & Applications*, vol. 4, no. 7, pp. 1201–1210, 2010.

[4] L. N. Egidio, H. R. Daiha, G. S. Deaecto, and J. C. Geromel, "DC motor speed control via buck-boost converter through a state dependent limited frequency switching rule," in *IEEE Conference on Decision and Control (CDC)*, 2017, pp. 2072–2077.

[5] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, "A survey of recent results in networked control systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 138–162, 2007.

[6] M. Camara, H. Gualous, F. Gustin, and A. Berthon, "Design and new control of DC/DC converters to share energy between supercapacitors and batteries in hybrid vehicles," *IEEE Transactions on Vehicular Technology*, vol. 57, no. 5, pp. 2721–2735, 2008.

[7] R. Shorten, F. Wirth, O. Mason, K. Wulff, and C. King, "Stability criteria for switched and hybrid systems," *SIAM Review*, vol. 49, no. 4, pp. 545–592, 2007.

[8] Z. Sun and S. S. Ge, *Switched Linear Systems Control and Design*. London: Springer-Verlag London Limited, 2005.

[9] D. Liberzon, *Switching in Systems and Control*. Boston: Birkhäuser Boston, 2003.

[10] M. Fiacchini, A. Girard, and M. Jungers, "On the stabilizability of discrete-time switched linear systems: Novel conditions and comparisons," *IEEE Transactions on Automatic Control*, vol. 61, no. 5, pp. 1181–1193, 2016.

[11] G. S. Deaecto, J. C. Geromel, and J. Daafouz, "Dynamic output feedback $\mathcal{H}_\infty$ control of switched linear systems," *Automatica*, vol. 47, no. 8, pp. 1713–1720, 2011.

[12] J. Lian, C. Li, and B. Xia, "Sampled-data control of switched linear systems with application to an f-18 aircraft," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 2, pp. 1332–1340, 2017.

[13] P. Bolzern and W. Spinelli, "Quadratic stabilization of a switched affine system about a nonequilibrium point," in *Proceedings of the 2004 American Control Conference*, vol. 5, 2004, pp. 3890–3895.

[14] G. S. Deaecto and G. C. Santos, "State feedback $\mathcal{H}_\infty$ control design of continuous-time switched affine systems," *IET Control Theory & Applications*, vol. 9, no. 10, pp. 1511–1516(5), 2015.

[15] G. S. Deaecto, "Dynamic output feedback $\mathcal{H}_\infty$ control of continuous-time switched affine systems," *Automatica*, vol. 71, pp. 44 – 49, 2016.

[16] C. Albea-Sanchez, G. Garcia, S. Hadjeras, W. Heemels, and L. Zaccarian, "Practical stabilisation of switched affine systems with dwell-time guarantees," *IEEE Transactions on Automatic Control*, vol. 64, no. 11, pp. 4811–4817, 2019.

[17] S. Baldi, A. Papachristodoulou, and E. B. Kosmatopoulos, "Adaptive pulse width modulation design for power converters based on affine switched systems," *Nonlinear Analysis: Hybrid Systems*, vol. 30, pp. 306–322, 2018.

[18] L. Hetel and E. Fridman, "Robust sampled–data control of switched affine systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 11, pp. 2922–2928, 2013.

[19] P. Hauroigne, P. Riedinger, and C. Iung, "Switched affine systems using sampled-data controllers: Robust and guaranteed stabilization," *IEEE Transactions on Automatic Control*, vol. 56, no. 12, pp. 2929–2935, 2011.

[20] G. S. Deaecto and J. C. Geromel, "Stability analysis and control design of discrete-time switched affine systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 4058–4065, 2017.

[21] L. N. Egidio and G. S. Deaecto, "Novel practical stability conditions for discrete-time switched affine systems," *IEEE Transactions on Automatic Control*, vol. 64, no. 11, pp. 4705–4710, 2019.

[22] G. S. Deaecto and L. N. Egidio, "Practical stability of discrete-time switched affine systems," in *2016 European Control Conference (ECC)*, 2016, pp. 2048–2053.

[23] L. N. Egidio, H. R. Daiha, and G. S. Deaecto, "Global asymptotic stability of limit cycle and $\mathcal{H}_2/\mathcal{H}_\infty$ performance of discrete-time switched affine systems," *Automatica*, vol. 116, p. 108927, 2020.

[24] W. Chen, Y. Liu, X. Li, T. Shi, and C. Xia, "A novel method of reducing commutation torque ripple for brushless DC motor based on cuk converter," *IEEE Transactions on Power Electronics*, vol. 32, no. 7, pp. 5497–5508, 2017.

[25] M. Frank and P. Wolfe, "An algorithm for quadratic programming," *Naval Research Logistics (NRL)*, vol. 3, no. 1-2, pp. 95–110, 1956.

[26] S. Boyd, E. Ghaoui, L.E. nd Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, ser. Studies in Applied Mathematics. SIAM, 1994.